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A tree-projection-based algorithm for multi-label recurrent-item associative-classification rule generation

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Abstract

Associative-classification is a promising classification method based on association-rule mining. Significant amount of work has already been dedicated to the process of building a classifier based on association rules. However, relatively small amount of research has been performed in association-rule mining from multi-label data. In such data each example can belong, and thus should be classified, to more than one class. This paper aims at the most demanding, with respect to computational cost, part in associative-classification, which is efficient generation of association rules. This task can be achieved using different frequent pattern mining methods. In this paper, we propose a new method that is based on the state-of-the-art tree-projection-based frequent pattern mining algorithm. This algorithm is modified to improve its efficiency and extended to accommodate the multi-label recurrent-item associative-classification rule generation. The proposed algorithm is tested and compared with A priori-based associative-classification rule generator on two large datasets.

Keywords: Association rules; Associative classification; Tree projection; Multi-label rules; Recurrent-item rules

1. Introduction

Classification aims at assigning labels to objects. First a classifier(s) is built based on a set of training objects for which labels are known, and next the classifier is used to recognize previously unseen objects and assign labels to them. The process of building classifiers (called learning) as well as the process of classification itself can be realized in many different ways using variety of machine learning techniques such as support vector machines, naive Bayes, instance-based classification, decision trees, etc. Our research addresses *associative-classification* which is a relatively new classification method that has been recently gaining researchers' attention. The advantage of using associative classification lies in (1) the simplicity of the core idea, i.e., using a statistical approach (finding frequent patterns) may be perceived as conceptually simpler and more intuitive than creating a mathematical model (such as in SVM or Naïve Bayesian classifiers) yet the technique gives comparable accuracy as opposed to even simpler but less accurate instant-based methods, and (2) the

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descriptive nature of the model, i.e., a flat list (as opposed to, e.g., decision trees) of human-interpretable and independent (modular) rules.

Several different associative-classification methods have been proposed so far, such as CBA [20], CMAR [19], CPAR [35], ARC-AC/BC [36], L_3^M [3], or MMAC [34,33], as well as some modification to a standard classification task such as CAEP [8], a classifier that captures changing trends in data over time. However, the majority of research work on associative-classification deals only with single-label classification, i.e., assigning only one class label to an object. Although multi-label classification, i.e., assigning one or more class labels to an object, has been widely studied, e.g., in [18,4,21,22,10], relatively small amount of work has been devoted to multi-label *associative* classification. Multi-label classification is desired in fields such as text categorization. For instance, article references from the MEDLINE database, consisting of over 13 million records, are being assigned by National Library of Medicine's (NLM) employees to, in most cases, over ten different Medical Subject Headings (MeSH). A single-label classifier would be obviously of no avail in this case.

One of the commonly used methods to deal with multi-label classification is to use rules generated during a standard single-label rule mining and employ *thresholding* techniques to choose more than one rule matching a new and unknown object during classification. That approach, with respect to associative-classification, has been already discussed in [31]. In this paper, we present an algorithm that generates rules each of which is associated with one or more labels and thus may be sufficient to cover a new unknown object. We also show that multi-label rules being supersets of single-label rules can be generated with relatively low cost, when compared to single-label rule generation, introducing a set of additional, to single-label, rules. Moreover, the proposed method considers re-occurrence of features in a single object in both learning and classification process as described in [32].

The method is based on the state-of-the-art frequent pattern mining algorithm described in [2]. Although this paper is focused on efficient *generation* of association rules, we also show a classification schema built on generated rules. The accuracy of the classifier is evaluated and compared to decision-tree-based, rule-based, and two other associative classifiers.

The next section defines a problem of associative-classification and presents current research in the field. Section 3 describes the design of the proposed algorithm including frequent pattern mining algorithm and a multi-label recurrent-item associative-classification rule generation algorithm. The results of performed experiments are shown in Section 5, whereas Section 6 summarizes the paper.

2. Background

2.1. Problem definition

We start by defining a problem of multi-label recurrent-item associative-classification rule generation.

Let $\mathscr{C}$ be a set of labels and $\mathscr{I}$ a set of items. The dataset D consists of transactions being a powerset of $\mathscr{C}$ and $\mathscr{I}$ in the form of $\langle X_i, C_i \rangle$ where $X_i \subset \mathscr{I}$ is a set of items $\{x_{i1}, x_{i2}, \ldots, x_{ik}\}$ and $C_i \subset \mathscr{C}$ is a set of labels $\{c_{i1}, c_{i2}, \ldots, c_{ij}\}$ for each transaction T_i with j labels and k items, such that $\bigcup_{i=1}^{n} T_i = D$.

The task of associative-classification rule generation is to find association rules in the form of $X \Rightarrow C$ indicating strong relationship between items in X and the set of classes C. (Traditional associative-classification considers C as a single class as opposed to a multi-label scenario presented in this paper.) The set of items X in a rule is commonly called *condition set* or simply *condset*.

There are two measures indicating the strength of a rule. The *support* σ of the rule $X \Rightarrow C$ is the fraction of transactions in D that contain both X and C, or formally:

$$\sigma_D(\langle X, C \rangle) = \frac{\hat{\sigma}_D(\langle X, C \rangle)}{|D|} \tag{1}$$

where $\hat{\sigma}_D(x)$ denotes the number of occurrences of x in D.

The *confidence* φ of the rule is the fraction of transactions containing X which also contain C, or formally:

$$\varphi_D(\langle X, C \rangle) = \frac{\hat{\sigma}_D(\langle X, C \rangle)}{\hat{\sigma}_D(X)} \tag{2}$$

In text categorization documents and words they contain are equivalent to transactions and items, respectively, as defined above. In the remaining part of this paper we use these terms interchangeably.

Recurrent-item association-rule mining is one of the extensions to association-rule mining known from [1]. The information on the number of occurrences of each item in a single transaction has been applied to Associative-Classification in [32] and comparison to the "non-recurrent" approach has been presented in [31]. In recurrent-item associative-classification transactions are in the form of $\langle\{\rho_1 x_1,\ldots,\rho_n x_n\}, C\rangle$, where $x_i \in \mathscr{I}$ is an item, $C \subset \mathscr{C}$ is a set of labels, and ρ_i is the number of occurrences of the item x_i in the transaction.

Let $T = \langle X_T, C_T\rangle$ be a transaction such that $X_T = \{\rho_{1T}x_1,\ldots,\rho_{mT}x_m\}$ and $C_T = \{c_1,\ldots,c_n\}$, and $R = \langle X_R, C_R\rangle$ be a ruleitem such that $X_R = \{\rho_{1R}x_1,\ldots,\rho_{kR}x_k\}$ and $C_R = \{c_1,\ldots,c_l\}$. We say that transaction T *supports* ruleitem R if $\forall i \in [1,l]: c_i \in C_R \Rightarrow c_i \in C_T$ and $\forall j \in [1,k]: x_j \in X_R \Rightarrow x_j \in X_T \wedge \rho_{jR} \leqslant \rho_{jT}$. In other words, a transaction supports a ruleitem if each item and label from the ruleitem has its correspondent in the transaction and the number of occurrences of each corresponding item in the transaction is no less that this in the ruleitem.

A simple example enhancing the difference between recurrent- and non-recurrent-item representation is shown in Fig. 1. Recurrent-item representation allows for further discrimination of the rules based on the number of recurrent items in both the document and rules. In the given example, both rules R_1 and R_2 in non-recurrent-item representation match document D, however, when item recurrence is considered, R_2 no longer matches D due to an excess amount of item b.

2.2. Related work

Associative-classification stems from *association-rule mining*, a data mining technique introduced in [1]. Integration of association-rule mining with classification has been shown in the CBA algorithm [20]. The authors extended the commonly used A priori algorithm [1] to generate classification rules. Similar approach has been employed in the family of associative classification algorithms ARC-AC/BC [36]. However, A priori-based algorithms, that use *candidate set generate-and-test approach* are computationally expensive, especially with long and numerous patterns in an input dataset. As opposed to the previous algorithms, CMAR [19] uses the *FP-growth* approach based on an *FP-tree* [14,15], which is claimed to be an order of magnitude faster than the A priori algorithm.

Another approach to accelerate rule generation and improve classification accuracy has been proposed in CPAR [35], an algorithm that integrates rule-based methods, such as FOIL/FFOIL [27,29,30,28] and RIPPER [5,6], to generate rules with features of associative-classification in predictive rule analysis.

A technique based on *intersection method*, presented in [37], has been proposed in MCAR [34] and MMAC [33].

However, researchers focus their attention more on classification schemata based on previously generated rules (using well known techniques) than on efficient generation of those rules, which is fully justified providing that the created algorithms work with small datasets. Our work, in contrast, aims at the efficiency of rule generation. Additionally, as opposed to the above mentioned papers that discuss single-label classification and

unassigned document D: $\{a,b,c,d,a,b,c,a,c,a,c\}$

non-recurrent-item representation	*recurrent-item* representation
$D=\{a,b,c,d\}$	$D=\{4a,2b,4c,1d\}$
rules	rules
$R_1=\langle\{a,b,d\},\{C_1,C_2\}\rangle$	$R_1=\langle\{3a,2b,1d\},\{C_1,C_2\}\rangle$
$R_2=\langle\{a,b,c\},\{C_2,C_3\}\rangle$	$R_2=\langle\{3a,3b,2c\},\{C_2,C_3\}\rangle$
Both R_1 and R_2 match D	Only R_1 matches D ($\{3a,3b,2c\} \not\subset D$)

Fig. 1. Difference between *non-recurrent*- and *recurrent*-item representation.

are tested on relatively small datasets, our approach aims at multi-label recurrent-item classification with applications to large datasets (containing hundreds of thousands of transactions). Although MCAR and MMAC deal with multiple labels, they generate rules by iteratively repeating the adopted method for generating single-label rules. At the same time, our algorithm generates multi-label rules in a *single* run.

We adopt and modify very efficient *dataset-projection-based* method that can be easily applied to other association-rule algorithms, such as the A priori or *pattern-growth* algorithms such as FP-growth. However, applying recurrent items into the FP-tree, used in the FP-growth algorithm, appeared to be a difficult task. To the best of our knowledge only one such approach has been published so far [24], but the method does not discover all frequent patterns. The A priori algorithm is more flexible in this matter because it does not impose restrictions on how data are stored.

Currently there are several techniques that perform efficient projection of the data to generate association rules. A pattern-growth approach [12], adopts a *divide-and-conquer* method to project and partition the dataset based on the currently discovered patterns. This method has been applied in, e.g., FP-growth [14], FreeSpan [13], PrefixSpan [26,25], and a framework for parallel data mining [7]. Similar divide-and-conquer approach has been applied in a *tree-projection* algorithm [2], where frequent itemsets and their projected datasets are embedded in a tree structure. The same tree structure has been further used in *sequential pattern mining* [11].

The method proposed in this paper is based on association-rule mining which can be treated as an extension to frequent pattern mining, i.e., the structure of discovered frequent patterns is divided into precedent and consequent to form a rule $X \rightarrow C$. Moreover, when considering associative-classification X has to represent a set of object's features (items) and C has to represent a set of labels describing the object.

The core of the proposed algorithm, i.e., discovering frequent patterns (itemsets), is similar to the tree-projection algorithm described in [2] in that information about itemsets together with their projected datasets are organized in a tree structure. Main differences lie in that the branches of our proposed tree represent only particular items instead of the whole itemsets, and that new branches are created directly from the tree without using additional structures (which require additional space) such as triangular matrices used in [2]. Such structure allows for significant reduction of the number of candidate tests, which is a crucial problem for association-rule algorithms producing candidates to obtain longer itemsets. Furthermore, our algorithm uses less space to store the projected datasets. In order to accommodate the algorithm to deal with class labels (or more precisely with multiple class labels) as well as with re-occurrence of items in a single transaction further modifications have been imposed on the projected tree, as shown in the next section.

To recapitulate, our work aims at the efficient generation of rules to fill the gap between qualitative aspects of associative-classification, that have been gaining researchers' attention for several years now, and quantitative capabilities of such classifiers. Additionally, we introduce recurrent items and *instantaneous* generation of multiple labels in associative-classification.

3. Design

This section discusses the design of the proposed algorithm and is organized as shown in Fig. 2. The section starts with introducing frequent pattern mining problem and its extensions and optimizations which are applied in the proposed associative-classification rule generation algorithm. Section 3.1 presents the problem of frequent pattern mining using the tree-projection technique. Although frequent pattern mining is not directly addressed in this paper, associative-classification rule generation is based on this approach. Sections 3.2–3.5 discuss the extensions and constraints that need to be applied to frequent pattern mining in order to obtain the algorithm for associative-classification rule generation. The final algorithm has two possible flavors that are based on breadth-first and depth-first search algorithms. These are presented in Sections 4.1 and 4.2, respectively.

3.1. Frequent pattern mining algorithm

3.1.1. Tree-projection

The proposed algorithm is based on a tree consisting of nodes representing items and labels connected by edges in a way the tree becomes a graphical representation of rules.

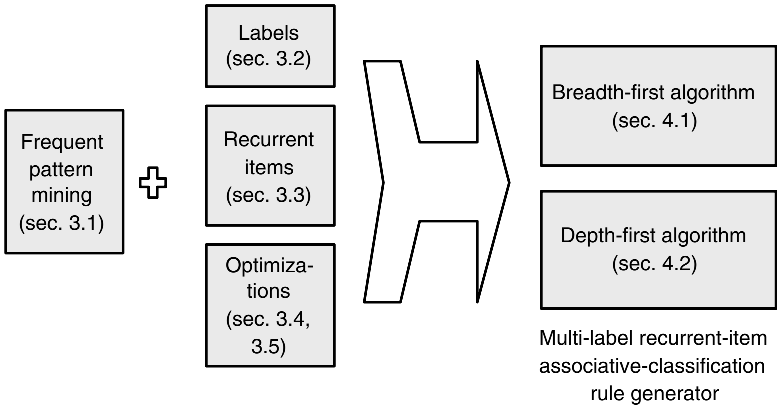


Fig. 2. Design overview.

To improve transparency of the description we initially describe the problem without considering class labels, focusing on items only. The tree discussed in this section is therefore called an *itemset tree* in contrast to a *ruleitem tree* which is discussed later on when labels are added (see Section 3.2).

Following the problem defined in Section 2.1 let us further assume that there is an order between items, e.g., based on the position of items in the input dataset, that is kept for each transaction. Expression $x_i < x_j$ denotes that item x_i precedes x_j. The itemset tree is defined as follows:

1. Each vertex (node), except the root node, in the tree represents an item in $\mathcal{I}$.
2. Each edge corresponds to an order between two items in a transaction.
3. The root vertex does not correspond to any item and does not have any incoming edges.
4. Each path of length l in the tree connecting a root vertex and any other vertex corresponds to an l-itemset, either frequent or hypothetical (candidate), such that each vertex in the path represents a single item in the itemset.

An example of an itemset tree and a corresponding set of itemsets is shown in Fig. 3. This tree is complete, i.e., it consists of all possible itemsets that can be created from four items. The itemsets shown in this example correspond to paths spanning from the root of the tree to its nodes. For instance, the longest itemset $\{1,2,3,4\}$ is represented by the left-most path spanning from the root, through nodes 1, 2, and 3 to node 4.

For simplicity, given node p and item x represented by p, we refer to p as if it was actually item x, i.e., $x \in \mathcal{I} \Rightarrow p \in \mathcal{I}$. For any two nodes p and q, expression $p = q$ denotes that both represent the same item.

We provide several definitions that facilitate further descriptions as follows:

Level: Given node p other then the root, *level* l of p is the length of the path from the root node to the node p. $V(l)$ denotes a set of nodes at level l.

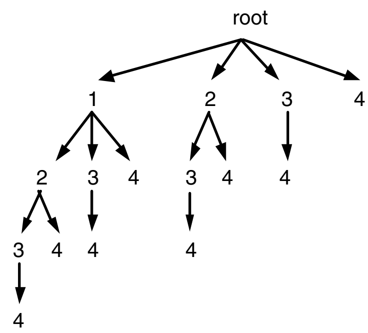
Parent: A node is called *parental node* or *parent* if it is a seed to generate next level of nodes such that each of newly generated nodes has exactly one edge connecting it to the parent. Given node q, its parent is denoted by $P(q)$.

Candidate: A node generated from another node is called *candidate*. $G(p)$ denotes a set of candidates outgoing (branching) from the same parent p.

Frequent node: Node q generated from node p is frequent if $q \in G(p)$ and q satisfies the support threshold ξ. $F(p)$ denotes a set of frequent nodes generated from p, and $\forall p\ F(p) \subset G(p)$.

Successor: Given node q, each node s_i that succeeds q, i.e., $q < s_i$, and has the same parent p is called *successor*. $S(q)$ denotes a set of successors of node q. For any level l, $\forall q \in V(l) \iff S(q) \subset V(l)$.
The tree depicted in Fig. 3 is *complete*, i.e., support threshold $\xi = 0$, which means that $F(p) = G(p)$ for every node p.

a



b

1-itemsets: $\{1\}, \{2\}, \{3\}, \{4\}$

2-itemsets: $\{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}$

3-itemsets: $\{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}$

4-itemset: $\{1,2,3,4\}$

Fig. 3. Example: (a) itemset tree and (b) corresponding itemsets.

For reader's convenience the terminology is recapitulated in Table 1.

The outcome of an algorithm for generation of frequent itemsets (an algorithm for generation of associative-classification rules is presented later in this paper in Section 3.2) is the itemset tree consisting of frequent nodes only. The nature of the algorithm is to generate nodes beginning from the root (bottom-up strategy). The approach is similar to the A priori algorithm [1] in that it produces candidate nodes that are tested against a set of transactions. However, unlike A priori the set of transactions for testing k-itemsets is limited to the transactions consisting of $k-1$-itemsets which means that each node p in the tree has its own *projected dataset*.

Projected dataset: For each node q and its parent $p = P(q)$ there is a set $D(q) \subset D(p)$, called *projected dataset*, such that $q \in T_i$ for each transaction $T_i \in D(q)$ and for the root node r $D(r) = D$.

3.1.2. Basic algorithm

This section describes the general behavior of the algorithm with respect to generation of frequent itemsets. Figs. 4–7 show the pseudocode of the algorithm. The following description is a high level explanation of algorithm's mechanisms that does not fully reflect the actual implementation. The implementation contains several

Table 1
Terminology

$\sigma, \hat{\sigma}$	Support and support count, respectively
$\xi, \hat{\xi}$	Support threshold and support threshold count, respectively
ρ	Number of word occurrences
$V(l)$	Set of nodes at level l
$G(p)$	Candidates originated from node p
$F(p)$	Frequent nodes originated from node p
$S(p)$	Successors of node p
$P(p)$	Parent of p

```
   // r - root node
1  G(r) ≡ V(1) := I
2  CountSupport(G(r))
3  F(r) := Prune(G(r))
4  l := 1
5  while V(l) ≠ ∅ do
6      for each p ∈ V(l) do
7          G(p) := GenerateCandidates(S(p))
8          CountSupport(G(p))
9          F(p) := Prune(G(p))
10     end
11     l = l + 1
12 end
```

Fig. 4. General algorithm.

```
Input: set of successors S
Output: set of candidates G
for each q ∈ S do
    G := G ∪ q
end
```

Fig. 5. `GenerateCandidates()` function.

```
Input: set of candidates G
Input: projected dataset D_p
for each q ∈ G do
    for each T ∈ D_p do
        if q ∈ T then
            σ̂(q) := σ̂(q) + 1
            D(q) := D(q) ∪ T
        end
    end
end
```

Fig. 6. `CountSupport()` procedure.

significant optimizations which are not presented here for better readability. The implementation details are discussed in Sections 3.4 and 3.5.

Given a set of all possible items $\mathscr{I}$ in dataset D, the first level of nodes is generated based on the frequency of the occurrences of these items in the dataset (Fig. 4). Procedure `CountSupport()` in line 2 goes through

```
Input: set of candidates G
Input: support count threshold ξ̂
Output: set of frequent nodes F
F := G
for each q ∈ G do
    if σ̂(q) < ξ̂ then
        F := F \ q
    end
end
```

Fig. 7. `Prune()` function.

the dataset and increases support count for each node from the set passed to the function as an argument. Infrequent nodes are pruned (line 3) based on support threshold. Subsequent levels are created based on preceding levels. The three functions `GenerateCandidates()`, `CountSupport()`, and `Prune()` (lines 7–9) are repeated for each node at the current level. (Note that nodes $V(l)$ for $l > 1$ may have different parents.) The algorithm stops if there is no new level of nodes to generate further nodes from (the condition in line 5).

Details of function `GenerateCandidates()` are shown in Fig. 5. The function simply creates a copy of a node and ties it with an appropriate parent.

Procedure `CountSupport()`, shown in Fig. 6, needs more attention. The existence of each candidate is verified against a projected dataset (lines 2–7). If a candidate exists in a currently processed transaction T (line 3), its support count is increased by one (line 4), whereas T is added to candidate's projected dataset $D(q)$ (line 5). Therefore the function not only counts support but also creates projected datasets.

Function `Prune()`, shown in Fig. 7, simply verifies the support count of each candidate and prunes those that do not satisfy a given support threshold.

The actual implementation counts the support and creates projected datasets using more advanced techniques. More specifically transactions are verified against the whole set of candidates $G(p)$ at once and projected datasets keep only references to transactions in dataset D which is discussed in details in Section 3.4.

The following section addresses adding class labels into the rule generation process.

3.2. Labels in the projected tree

This section focuses on building the complete rule tree, i.e., a tree containing both items and labels. The easiest way to include labels in a frequent itemset is to treat them as items, i.e., neglect distinction between items and labels. However, this solution would result in a vast amount of association rules without a label and rules containing labels only. In other words, there would be cases where creating a rule in the form of $X \Rightarrow C$ would be impossible due to the lack of either X or C. Although after pruning incomplete rules this solution is undoubtedly correct, it is highly inefficient. For instance, in the tree shown in Fig. 3 if "1" denotes a label and "2", "3", and "4" denote items, from the total of 15 rules only seven have a label and at least one item. Using real datasets and $\xi > 0$ the number of complete rules, $X \Rightarrow C$, is significantly smaller.

To avoid generation of unwanted rules both the tree and algorithm need to be modified. Similarly to items, we assume that there is an order between labels. We propose to implement the following constraints into the tree to efficiently generate association rules with class labels:

- The first level consists of nodes representing labels only.
- The second level consists of nodes representing items only.
- Levels three and higher consist of nodes representing both items and labels.
- If nodes have the same parent, nodes representing items precede those representing labels.

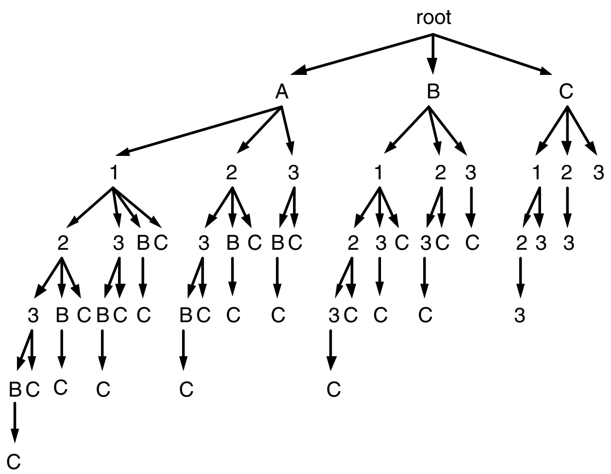


Fig. 8. Multi-label ruleitem tree.

An example of a complete tree consisting of items and labels is depicted in Fig. 8. Labels and items are denoted by capital letters and numbers, respectively.

Situating labels at the first level ensures that all rules have at least one label. Lack of labels at the second level prevents from generating rules without items. Note that although the first level of labels is essential for building further generations, it cannot be used to produce any rules by itself.

The algorithm for frequent itemset mining presented in the previous section must be modified to accommodate for the class labels. However, it still can be used applying some modifications without giving up the intrinsic characteristics of the algorithm. We observe that the second level candidates cannot be generated from successors $S(p)$ of any first-level node p. Similarly the third level of nodes does not fully follow `CandidateGeneration()` function. However, generation of the nodes of levels greater then three does comply with the function. This leads to the modifications of the basic algorithm as shown in Fig. 9.

The new algorithm consists of four parts. The first part builds the first level of nodes representing labels (lines 1–3). The second part (lines 4–8) assigns items to each first-level node. At this point it is possible to produce fully qualified rules, $X \Rightarrow C$, consisting of exactly one item and one label.

Up to now each level greater than one is built based on its preceding level only. However, the generation of the third level involves two preceding levels. Given node p at level two, the third level candidates are generated based on successors $S(p)$ as well as successors of p's parent $S(P(p))$ (line 10). Levels four and higher are generated in the same fashion as described in previous section (see Fig. 4).

3.3. Recurrent items

Recurrent items have been already defined in Section 2.1. A fragment of a tree consisting of recurrent items and its corresponding set of ruleitems is shown in Fig. 10. To avoid ambiguity between item's identifier and a number of its occurrences, the latter one is put in round brackets, i.e., notations $\rho_i x_i$ and $x_i(\rho_i)$ are equivalent.

The algorithm is adapted to take recurrent items into consideration by introducing one modification: a set of successors $S(p)$ of node p is extended by this node. To further limit the number of candidates the following constraints must also be applied. For a given node p, for which candidate branches are to be created:

- if $p \in \mathscr{C}$ then p must not be added to the list of successors, and
- if $p \in \mathscr{I}$ then p is added to the list of successors if and only if the number of p's ancestors representing the same item in the path from the root to p is less that the maximum number of occurrences of p per transaction in a dataset.

```
// r - root node
G(r) ≡ V(1) := C
CountSupport(G(r), D(r))
Prune(G(r), ξ̂)
for each p ∈ V(1) do
    G(p) := I
    CountSupport(G(p), D(p))
    F(p) := Prune(G(p), ξ̂)
end
for each p ∈ V(2) do
    G(p) := GenerateCandidates(S(p) ∪ S(P(p)))
    CountSupport(G(p), D(p))
    F(p) := Prune(G(p), ξ̂)
end
l := 3
while V(l) ≠ ∅ do
    for each p ∈ V(l) do
        G(p) := GenerateCandidates(S(p))
        CountSupport(G(p), D(p))
        F(p) := Prune(G(p), ξ̂)
    end
    l = l + 1
end
```

Fig. 9. General algorithm for mining ruleitem tree.

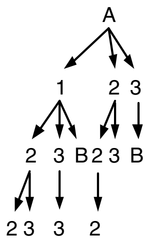
Indeed, labels in a rule need to be distinct, and the maximum number of occurrences of any item in a ruleitem cannot be more than the maximum number of occurrences of that item in any transaction. The latter limitation requires the algorithm to store the information about a maximum number of occurrences of items. This can be simply achieved by the initial reading of a dataset when the algorithm builds the index of frequent items.

3.4. Candidate test optimization

Most of the solutions related to the generate-and-test approach of frequent itemset mining rely on very simple candidate test method. Each candidate itemset is verified against each transaction from a projected (or original) dataset.

Let $X = \{x_1, x_2, \ldots, x_k\}$ be a candidate k-itemset and $T = \{y_1, y_2, \ldots, y_l\}$ be a transaction of length l. The easiest strategy to test the candidate would be to compare each item in X with every item in T. In the worst case scenario this results in $k \times l$ comparisons. If items in both X and T are ordered (the order in X is embedded into the generated tree, whereas the order in T requires a single sort operation) then the complexity decreases to the size of either a transaction or itemset, whichever is longer (in most real-life cases transactions are longer than itemsets). This, however, has to be repeated $|D(p)|$ times for every node p in the tree.

Instead of testing each itemset against every transaction we can again use the generated ruleitem tree together with the projected datasets. Maintaining projected datasets allows for checking only the last item a



b

2-ruleitems: $\langle\{1(1)\},\{A\}\rangle,\ \langle\{2(1)\},\{A\}\rangle,\ \langle\{3(1)\},\{A\}\rangle$

3-ruleitems: $\langle\{1(1),2(1)\},\{A\}\rangle,\ \langle\{1(1),3(1)\},\{A\}\rangle,$
$\langle\{1(1)\},\{A,B\}\rangle,\ \langle\{2(2)\},\{A\}\rangle,$
$\langle\{2(1),3(1)\},\{A\}\rangle,\ \langle\{3(1)\},\{A,B\}\rangle$

4-ruleitems: $\langle\{1(1),2(2)\},\{A\}\rangle,\ \langle\{1(1),2(1),3(1)\},\{A\}\rangle,$
$\langle\{1(1),3(2)\},\{A\}\rangle,\ \langle\{2(3)\},\{A\}\rangle$

Fig. 10. Fragment of (a) tree with *recurrent items* and (b) its corresponding set of ruleitems.

in an itemset represented by a node in $G(p)$ for some p, since the remaining part of the itemset has been already verified in the previous levels. We also observe that $G(p)$ is ordered in the tree. Therefore we can test the entire $G(p)$ against a transaction at once. However, now instead of testing whether $X \subset T$, the support of each item $x_i \in X$ is increased if $x_i = y_j$ for any $y_j \in T$. Thus, our approach results in $|G(P(p))|$ times smaller number of operations.

Procedure `IncreaseSupportAndCreateProjDatasets()`, shown in Fig. 11, performs the candidate test described above. For any given set X, function `Next(`X`)` successively returns items from set X, one at each call, with respect to order in X.

3.5. Dataset optimization

The algorithm distinguishes between two types of datasets: generic and projected. Generic dataset is read from a hard drive and stored in main memory. This dataset is kept at the root node of the ruleitem tree and used for a candidate test at the first level of the tree. Projected datasets are obtained from the generic dataset and used for candidate test at levels higher then one. This section discusses the optimization techniques for storage and accessing both generic and projected datasets.

There are several different techniques of storing a dataset in the main memory. One possibility is to keep transactions in the form of bit vectors so that each bit represents either the existence or absence of an item. This allows for very fast candidate test if itemsets are kept in the same fashion. However, this solution is memory consuming, especially when dealing with sparse data which is the case, e.g., in text categorization.

Another solution is to keep items in a transaction in the form of a list. Although it slows down candidate tests, this structure is very often the only way to deal with large amount of sparse and highly dimensional data.

The proposed method uses item lists. To reduce the amount of required memory, the stored transactions consist of frequent items only. There are two methods to filter out the frequent items either by (1) reading the complete transactions once into the main memory and next pruning infrequent items or (2) reading the complete transactions to compute item frequency values and then reading the data once again and storing in the main memory only the frequent items which are selected based on the counts computed during the first

```
Input: set of candidates G
Input: transaction T
x := Next(T)
q := Next(G)
while x ∉ ∅ ∧ q ∉ ∅ do
    if x = q ∧ ρ(x) ≥ ρ(q) then
        σ̂(q) := σ̂(q) + 1
        D(q) := D(q) ∪ {T}
    else
        if x > q then
            q := Next(G)
        else
            x := Next(T)
        end
    end
end
```

Fig. 11. `IncreaseSupportAndCreateProjDatasets()` procedure.

reading. The first solution is faster because it reads the dataset only once. However, in some cases there might not be enough space to load the entire dataset.

Having the entire dataset in the main memory there is no need to copy transactions to the projected datasets for every node. Therefore the projected datasets keep only references to the generic dataset. This significantly reduces memory consumption.

Another optimization employed in the proposed algorithm is deleting projected datasets if they are no longer needed. If level $k+1$ has been created, the projected datasets of nodes at level k can be deleted.

4. The proposed algorithm

Two possible strategies can be used to explore the ruleitem tree either by using a breadth-first search or by using a depth-first search. These strategies differ from each other with respect to the total number of projected datasets required to compute the rules. The two strategies are discussed in the following sections.

4.1. Breadth-first strategy

In breadth-first search approach the tree is explored level by level, i.e., level $l+1$ of the tree is generated only if computations are completed for all nodes at level l. The basic mechanism of the breadth-first search approach has already been presented in the previous sections. Here we provide more details and discuss pros and cons of this approach.

Using the breadth-first search we can take advantage of an important property of the A priori algorithm: if an itemset is not frequent its superset is not frequent either. Due to the fact that the tree is a representation of itemsets, this property can be exploited to optimize the algorithm. Before going into details let us introduce a concept of a *base node*:

Base node: Given node p and q such that $q \in G(p)$, we call node $s \in S(p)$ *base node of* q, denoted $B(q)$, if node q was directly generated from node s.

In order for the algorithm to comply with the A priori property, before generating $(l+1)$-level node q for l-level node p from some successor $s \in S(p)$ it must verify whether node s belongs to frequent branches of p's

($l-1$)-level base node, i.e. whether $s \in F(B(p))$. This prevents from testing of nodes that are guaranteed not to be be frequent, and thus saves time needed for candidate test operations. The new procedure for generating candidates, namely `GenerateCandidatesBF()`, is depicted in Fig. 12.

The breadth-first algorithm, shown in Fig. 13, begins with determining a set of the first-level frequent nodes. After employing dataset optimization, described in Section 3.5, this set is equal to the set of labels $\mathscr{C}$, i.e., we

```
Input: node p
Input: set of successors S
Output: set of candidates G
for each s ∈ S do
    if s ∈ F(B(p)) then
        G := G ∪ {s}
    end
end
```

Fig. 12. `GenerateCandidatesBF()` function.

```
// Begin with root node r
F(r) ≡ V(1) := C
l := 1
while V(l) ≠ ∅ do
    for each p ∈ V(l) do
        if P(p) ∈ ∅ then
            G(p) := I
        else if P(p) ∉ ∅ ∧ P(P(p)) ∈ ∅ then
            G(p) := GenerateCandidatesBF(S(p) ∪ S(P(p)))
        else
            G(p) := GenerateCandidatesBF(S(p))
        end
        for each T ∈ D(p) do
            IncreaseSupportAndCreateProjDatasets(G(p))
        end
        F(p) :=Prune(G(p))
        Delete(D(p))
    end
    l = l + 1
end
```

Fig. 13. Breadth-first algorithm.

assume that $\mathscr{C}$ consists of frequent labels only. For each node in the current level l set of candidates $G(p)$ is generated (lines 5–11) following a procedure associated with level l of the tree (discussed in Section 3.2). Support for each candidate is calculated and candidates' projected datasets are created (lines 12–14) according to optimized procedure described in 3.4. Finally, the infrequent candidates are pruned and dataset $D(p)$ is deleted to release no longer needed portion of memory (lines 15–16). The procedure is repeated as long as newly generated frequent nodes exist.

4.2. Depth-first strategy

The main drawback of the breadth-first algorithm is that the number of projected datasets is equal to the number of nodes in the current level l plus one dataset at level $l-1$. For a large number of items and low support threshold values the tree may become very wide which may result in substantial memory consumption. A depth-first search approach addresses this problem. In this case branches are created path-wise, i.e., for a given node p a set of frequent nodes $F(p)$ is found and for each such node $q \in F(p)$ the procedure as for p is recursively repeated until the last level of the tree is reached.

After reaching the last level as shown in the example, the last but one is mined, etc.

A complete psuedocode of the proposed depth-first search algorithm is shown in Fig. 14.

Function `DepthFirst()` is performed for each frequent label. The function itself generates candidates following, again, the level-dependent procedure described in Section 3.2 (Fig. 14b, lines 3–12). Note that in depth-first strategy it is impossible to determine base node $B(p)$ for any given p during the process of generating candidates, simply because base node $B(p)$ is yet to be discovered. Therefore, `GenerateCandidates()` function does not follow `GenerateCandidatesBF()` depicted in Fig. 12. After support for each candidate is calculated, projected datasets are created, and infrequent candidates are pruned (lines 10–13), `DepthFirst()` function is performed recursively for each frequent candidate (lines 14–16). The memory occupied by the projected dataset can be released only after all frequent branches of the currently performed node are mined (line 17).

The maximum number of projected datasets held in main memory at the same time depends now on the number of levels and is equal to $\sum_{i=1}^{n}|G(p_i)|$ where n is the number of levels in the currently mined path. Note that $|G(p_i)| \leqslant |G(p_{i-1})|$.

The number of levels depends on the statistic features of a dataset and a given support threshold. The lower the threshold or the longer patterns in data, the bigger the number of levels and longer rules. Nevertheless, the maximum number of levels can never exceed the number of items, which gives a significant reduction in the number of projected datasets when compared to the exponential growth of projected datasets in the breadth-first strategy. A closer look at the complexity of these two algorithms is provided in the next section.

4.3. Complexity and limitations

This section discusses the complexity of the algorithms and their limitations. We compare our breadth-first and depth-first algorithms to each other as well as to our implementation of *tree-based* A priori (briefly described in Section 5).

4.3.1. Computational complexity

Runtime complexity depends on the number of nodes (candidates) in the tree and the computation time needed for each node. We address each of them separately and multiply the resulting complexities to obtain the total complexity. For simplicity (without loss of generality), let us treat labels in the same fashion as items.

The size of the tree depends on the number of items (and labels) and a *pruning factor*, i.e., the difference between the number of candidates and the number of frequent items. For simplicity of further analysis let us assume that this difference is constant for all groups of candidates, i.e., $G(p) - F(p) = k$ for each node p, where $k = \text{const}$.

This results in the following formula for the size of tree $\mathscr{T}$ (excluding the root node):

$$|\mathscr{T}_k(n)| = F_{k+1}(n+1) - 1 \tag{3}$$

```
// Begin with root node r
F(r) ≡ V(1) := C
for each f ∈ F(r) do
    DepthFirst(f)
end
// Definition of recursive function DepthFirst
Function DepthFirst(node p)
begin
    // Level-dependent candidate generation
    if P(p) ∈ ∅ then
        G(p) := I
    else if P(p) ∉ ∅ ∧ P(P(p)) ∈ ∅ then
        G(p) := GenerateCandidates(S(p) ∪ S(P(p)))
    else
        G(p) := GenerateCandidates(S(p))
    end
    for each T ∈ D(p) do
        IncreaseSupportAndCreateProjDatasets(G(p))
    end
    F(p) :=Prune(G(p))
    for each f ∈ F(p) do
        DepthFirst(f)
    end
    Delete(D(p))
end
```

Fig. 14. Depth-first algorithm.

where n is the number of items (and labels) and F_m is a generalized Fibonacci number, which, in its combinatorial representation [17], is:

$$F_m(n) = \sum_{i=0}^{\left\lfloor \frac{n+m-2}{m} \right\rfloor} \binom{n+m-2-(m-1)i}{i} \tag{4}$$

In the worst case scenario (e.g., a crude dataset containing only one transaction or a set of exactly the same transactions) the size of the tree is $2^n - 1$ (which can be derived from (3) for $k = 0$). This exponential growth becomes weaker for greater values of k, and the complexity can be expressed as $O(c^n)$ for some constant c, where $c = 2$ for $k = 0$ and converges to *1* with increasing values of k.

If, however, pruning factor k is relative to the number of items n (for instance, $k = n/2$, i.e., a situation where always a half of first-level candidates is frequent) it is expected that for certain relations of k and n the computational complexity will become lower than $O(c^n)$. Let us analyze the following recursive formula for the size of the tree:

$$|\mathscr{T}_k(n)| = \begin{cases} n & \text{if } n \leqslant k+1, \\ \sum_{i=1}^{n-k-1} |\mathscr{T}_k(i)| + n & \text{if } n > k+1, \end{cases} \tag{5}$$

This formula is derived directly from the visual representation of the tree (see Section 3). Eliminating the recursion in (5) results in the following formula dependent on the relation between k and n:

$$|\mathscr{T}_k(n)| = \begin{cases} n & \text{if } n \leqslant \frac{3k+1}{2}, \\ \frac{2n-3k-1}{2}k + n & \text{if } \frac{3k+1}{2} < n \leqslant 2k+1, \\ \sum_{i=1}^{n-2k-1} F_{k+1}(n-2k-i)i + \frac{2n-3k-1}{2}k + n & \text{if } n > 2k+1, \end{cases} \tag{6}$$

From (6) we can observe that the bigger the difference between n and k, the bigger the tree growth rate. With constant k and increasing n the size of the tree grows from $O(n)$ (the first case) to $O(n^2)$ (the middle case) to $O(c^n)$ (the latter case, where mainly the sum of generalized Fibonacci numbers dictates the growth). Thus, if k is relative to n the tree growth rate can decrease significantly. For instance, to continue with the previous example, for $k = n/2$ the tree growth becomes quadratic.

Let us now analyze the complexity of operations that need to be performed at each node of the tree. From the algorithms presented in Sections 3 and 4 we see that each node (a candidate) requires the following operations to be performed: (1) generation, (2) counting support, and (3) pruning. Generation is only a matter of adding a single item to the tree. The complexity of counting support was already discussed in Section 3.4 and is proportional to $\frac{|D(p)|}{|G(P(p))|}|T|$ for node p and transaction T, i.e., it takes $|G(P(p))|$ times less time than support counting in other algorithms. Pruning is only a simple *if* statement comparing two numbers and, if true, deleting the node.

The difference between the depth-first and breadth-first search algorithms, as described in Sections 4.1 and 4.2, lies in generating less candidates for the latter one (although it yields additional comparisons). However, the *asymptotic* complexity of those two algorithms remains the same. Thus, the majority of time is spent on the candidate test (counting support). The range of denominator $|G(P(p))|$ is 1 to n with lower values on deeper levels of the tree. However, numerator $|D(p)|$, bounded by $\hat{\xi}$ and $|D|$, also decreases at the same time. Although the asymptotic upper bound of the candidate test is still $O(m)$, where $m = |D|$, there is a substantial reduction in the number of candidate tests when compared to the algorithms of other researchers due to the decreasing size of projected datasets $D(p)$ and performing aggregated (vs. one node at a time) tests.

The total computational complexity of the proposed algorithms equals $O(nm)$, $O(n^2m)$, and $O(c^{nm})$ depending on the relation between k and n. It varies from being linear to quadratic to exponential (the first two only if the pruning factor depends on the number of items) with respect to the number of items (and labels) with the constant number of transactions, and linear with respect to the number of transactions with the constant number of items (and labels). Although the latter one is scalable, the tree-size-dependent complexity imposes certain limitations on our algorithms: they will not scale well in situations with long patterns in data (very similar transactions) or/and very small support thresholds. In both such situations the pruning factor will be close to zero, which, in turn, will "trigger" exponential growth in the number of operations. However, in some domains such as text categorization, the pruning factor is relatively high, and thus, the expected complexity should remain at most quadratic.

4.3.2. Memory

Due to the fact that our proposed algorithms keep projected datasets their memory consumption is obviously higher than, for instance, the one of A priori, which keeps only a dataset and the tree nodes in main memory, i.e., its memory consumption is proportional to $|D| + |\mathscr{T}|$. However, breadth-first and depth-first algorithms differ from each other in the number of projected datasets they keep (discussed in Sections 4.1 and 4.2). Thus, memory usage is proportional to:

- $|D| + |\mathscr{T}| + \sum_{p \in V_{\max}} |D(p)|$ for the breadth-first algorithm, and
- $|D| + |\mathscr{T}| + \sum_{i=0}^{\lfloor \frac{n+k}{k+1} \rfloor - 1} \sum_{q \in G(p_i)} |D(q)|$ for the depth-first algorithm,

where n, as before, is the number of items (and labels), p_i is an ith node of the currently searched path in the tree (with p_0 being the root node), and $V_{\max}$ is a set of nodes at the widest level of the tree with the following cardinality (assuming the concept of pruning factor as described in Section 4.3.1):

$$|V_{\max}| = \max_{0 \leqslant i \leqslant \lfloor \frac{n+k}{k+1} \rfloor} \binom{n+k-ki}{i}. \tag{7}$$

The above equation is obtained from (4) and is equal to the maximum component of the sum for $F_{k+1}(n+1)$ in (4), which can be estimated as $O(c^n)$ for constant c, $1 < c < 2$. The number of nodes in $G(p_i)$ for each p_i in the path is of $O(n^2)$. Based on these estimates and those made in Section 4.3.1 the asymptotic complexity, in terms of memory usage, can be written as:

- $O(m) + O(c^n) + O(c^n)O(m)$, and
- $O(m) + O(c^n) + O(n^2)O(m)$,

for the breadth-first and depth-first algorithms, respectively. Thus, the projected datasets increase memory usage (when compared to the A priori) exponentially and quadratically with the increasing number of items (and labels) for the breadth-first and depth-first algorithms, respectively.

5. Experiments

A number of experiments exploiting a wide space of parameter values and input data was performed to evaluate runtime and memory consumption of the proposed algorithm for the multi-label associative-classification with recurrent items.

We also implemented a priori-like version of associative-classification rule generator and compared it with the proposed algorithm. This algorithm works in similar fashion to the one described in [20]. The main difference is that it is based on the ruleitem tree as a ruleitem representation. We refer to this algorithm as *A priori*, whereas the two versions of the proposed algorithm, breadth-first and depth-first projected, are referred as *BF-TP* and *DF-TP*, respectively.

Although two measures indicating the strength of rules, namely support and confidence, are usually used as parameters of association-rule mining, only support is used in this paper. The confidence, though a very significant parameter, is usually often omitted (like in frequent pattern mining) because it does not directly influence the number of generated rules but is only used to prune some of them.

5.1. Experimental setup

Although many researchers demonstrate complexity of their algorithms using synthetic data or small single-label corpora [20,19,35,34], these data cannot be used in case of the proposed method. First of all, datasets used to verify frequent pattern mining methods lack labels which are an intrinsic part of associative-classification rule generation. Secondly, our intention is to primarily use this algorithm for text categorization. Therefore, we chose OHSUMED [16] and RCV1 [18] corpora which are standard benchmarks in the text categorization and, most importantly, provide multi-label transactions.

OHSUMED [16] is a corpus of 348,543 records extracted from the MEDLINE, National Library of Medicine's (NLM) database consisting of approximately 13 million article references to biomedical journal articles, limited to 5 years: 1987–1991. Each document is assigned to Medical Subject Headings (MeSH), NLM's controlled vocabulary thesaurus consisting of medical terms at various levels of specificity. We used a modified version of this collection described in [31]. The original collection was reduced to documents that have both title and abstract which resulted in the total of 233,445 documents. Over 22,000 headings arranged in 11-level structure were generalized to the second level resulting in the total of 114 class labels [31]. We refer to this dataset as *ohsumed-gen2*.

The second dataset, RCV1 [18], is much larger and contains 804,414 news stories from Reuters. From throughout several dimensions of labels available for this collection we chose the most commonly used and adequate for our goal, 103 labels of "topics". Each Reuters's news story is assigned to at least one topic that categorizes this story. We refer to this dataset as *rcv1-topics*.

Both datasets are summarized in Table 2.

The proposed algorithm is evaluated with respect to several performance measures such as runtime, memory consumption, and scalability. Several sets of experiments for various sizes of the datasets and support threshold values were performed, and both single- and multi-label rules were generated. The complete experimental setup is shown in Table 3.

The experiments were performed on a PC with a 2 GHz Pentium IV processor and 1.5 GB of RAM. Runtime is measured excluding the time of reading a dataset from a hard drive. The reading time depends on the size of a dataset only, i.e., it is indifferent to parameters and algorithms chosen, and ranges from several seconds for the smallest datasets used to several minutes for the biggest datasets.

5.2. Runtime

Fig. 15 shows the runtime values of the algorithms on the two datasets. Due to space limitations, runtime is shown only for datasets with 100,000 transactions and various support threshold values. Performance on the entire range of dataset sizes is analyzed in Section 5.4 that evaluates the scalability of the algorithm. Fig. 15a, c, e, and g shows the runtime in the function of a support threshold (note the reverse order). A priori is definitely slower than both BF-TP and DF-TP algorithms when exploring low values of the support threshold. We observe that the runtimes have tendency to rapidly grow with respect to the decreasing support. However, the number of rules generated grows nonlinearly with increasing value of the support threshold. Fig. 15b, d, f, and h shows the same runtime in function of the number of generated rules. The axes are scaled logarithmically to help visualize all points on the graph. The three algorithms show linear relationship between runtime and the number of rules. The dotted line on the graph is a linear approximation (in a least squares sense) for the DF-TP algorithm. Both BF-TP and DF-TP require significantly less runtime to generate the same number of rules when compared with A priori. The coefficients of linear approximations for the rcv1-topics dataset are $\{0.63, 114\}$, $\{0.019, 12\}$, and $\{0.021, 12\}$ for A priori, BF-TP, and DF-TP, respectively, where $\{a, b\}$ indicates coefficients for the linear approximation $f(x) = ax + b$.

Table 2
Dataset statistics

Dataset	Size	Number of labels			Number of words		
		Total	μ	σ	Total	μ	σ
ohsumed-gen2	233,445	114	9.8	3.1	99,775	95.6	40.62
rcv1-topics	804,414	103	3.2	1.4	288,062	123.9	110.3

μ, average per transaction; σ, standard deviation.

Table 3
Experimental setup

Dataset	Size (Transaction number) [k]	Support threshold [%]
ohsumed-gen2	12.5, 25, 50, 100, 200	2, 3, 4, 8, 12, 16, 20
rcv1-topics	25, 50, 100, 200, 400, 800	2, 3, 4, 8, 12, 16, 20

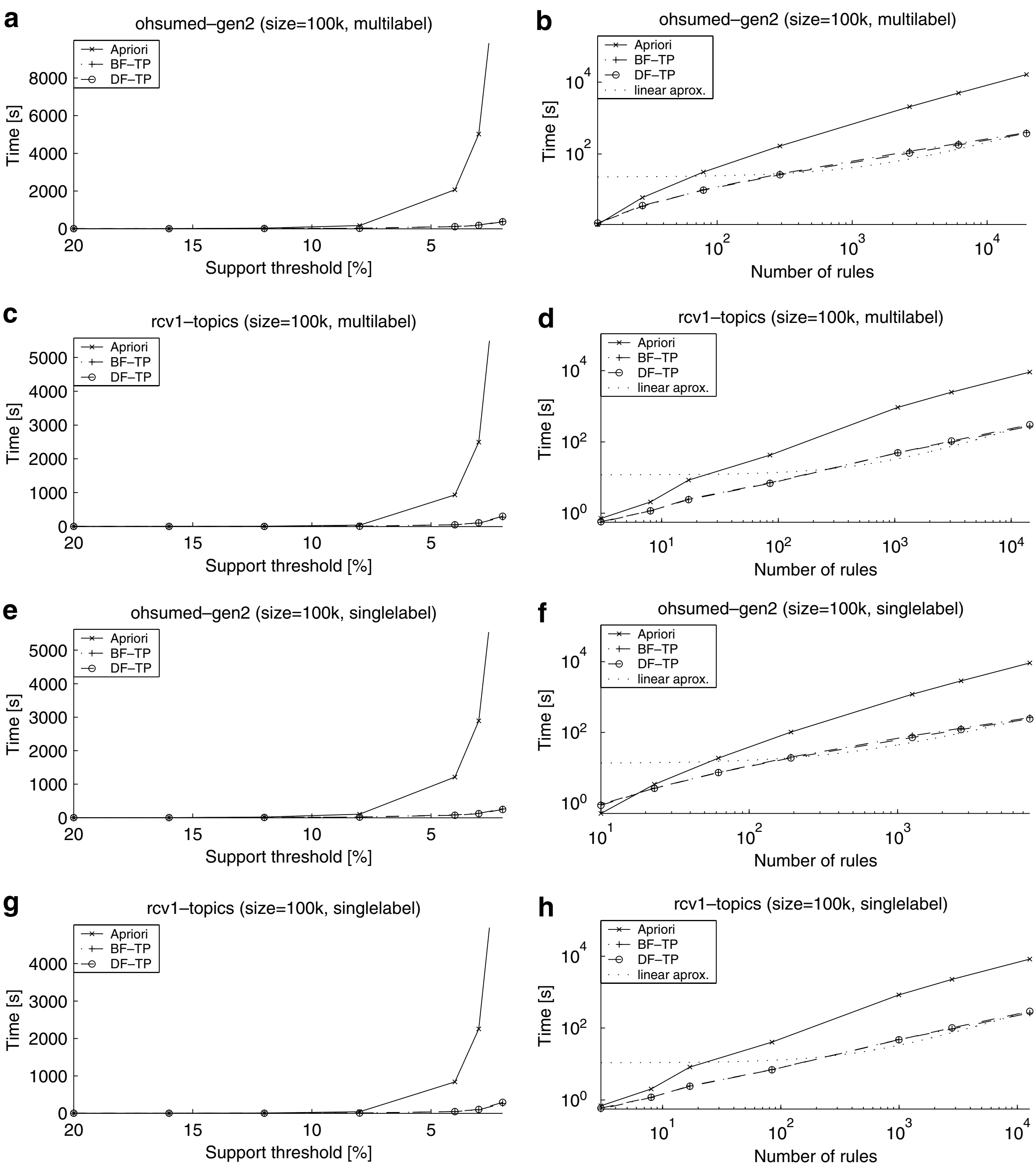


Fig. 15. Runtime.

Although the three considered algorithms are characterized by the same linear asymptotic complexity with respect to runtime, linear extrapolation to 50,000 rules shows that it takes almost 9 h to generate rules using A priori and only about 16 min using BF-TP.

The difference in runtime between A priori and the two proposed algorithms is a result of storing projected datasets in case of the two latter algorithms. The difference between BF-TP and DF-TP is most likely a result of a lower number of candidates in BF-TP. BF-TP limits this number during generation of nodes using information from upper levels of the tree, which is not available in case of DF-TP (for details refer to Section 4.1).

If we treat the decreasing support threshold as the way to increase the number of items in the mining process, we observe that the obtained results fully comply with complexity estimates ($O(c^n)$) elaborated in Section 4.3.1.

5.3. Memory consumption

Memory characteristics, shown in Fig. 16, are similar to those for the runtime, i.e., memory consumption rapidly grows with lower values of the support threshold (Fig. 16a, c, e, and g). However, in contrast to the runtime results, the algorithms performing faster, i.e., BF-TP and DF-TP, require more memory than the slowest, A priori. In fact, A priori's memory consumption changes only slightly even for a low support threshold when compared to BF-TP and DF-TP. Although BF-TP and DF-TP require comparable amount of memory for higher values of support threshold, the former one requires significantly more memory for medium and low values of support. Nevertheless, the three algorithms are characterized by approximately linear characteristic between memory consumption and the number of rules, as shown in Fig. 16b, d, f, and h.

Since A priori does not store projected datasets, the size of memory it requires is dependent only on the number of generated rules. In case of both BF-TP and DF-TP relatively large memory consumption is due to storing projected datasets. However, we note that DF-TP consumes significantly less memory than BF-TP when generating a large number of rules. For DF-TP, the number of projected datasets depends on the depth of the tree and not, as in case of BF-TP, on its width. As a consequence, since lowering support threshold results in creation of wider, rather than deeper, ruleitem trees, DF-TP is characterized by worse memory consumption when compared to BF-TP.

Again, treating the decreasing support threshold as the way to increase the number of items, the obtained results confirm our estimates made in Section 4.3.2. It also confirms the limitations of our algorithms: either a low support threshold or long patterns in data, i.e., the situation where very similar transactions yield a large number of patterns even if a support threshold is high, will result in numerous nodes in the tree and potentially large projected datasets (or rather references to the generic dataset as discussed in Section 3.5) on those nodes. This, in turn, may result in memory overflow. Nevertheless, setting a reasonable support threshold is left to user's disrection (e.g., buliding a rule-based classification model that consists of more rules than transactions may raise some doubts on usefullness of such a model) and becomes an engineering task itself.

5.4. Scalability

Scalability test was performed by iteratively increasing the number of transactions in the input dataset and measuring the corresponding results. The results are shown in Fig. 17. Both runtime and memory consumption indicate linear relation with the growing number of transactions (as estimated in Sections 4.3.1 and 4.3.2). Similarly to the previous results, BF-TP and DF-TP show comparable scalability and are superior to A priori with respect to runtime (Fig. 17a, c, e, and g). For instance, for the multi-label rcv1-topics dataset and the support threshold of 4% it takes about 237 seconds for A priori to generate rules from 25,000 transactions and about 12 s for DF-TP. For larger 800,000-transaction dataset A priori needs almost two hours compared to about 7 min for DF-TP.

On the other hand, memory consumption diagrams (Fig. 17b, d, f, and h) shows the opposite relation. Memory usage for A priori is almost constant, whereas for TP-algorithms, the required memory is linearly increasing with the increasing number of transactions.

Assuming uniform distribution of words in a dataset, the number of rules generated with the same support threshold should be virtually the same for any subset of the dataset. This implies that the ruleitem tree structure be almost identical. That is why A priori use nearly the same amount of memory for different dataset sizes. Since TP-algorithms need to keep projected datasets, their memory consumption increases linearly with the size of the generic dataset.

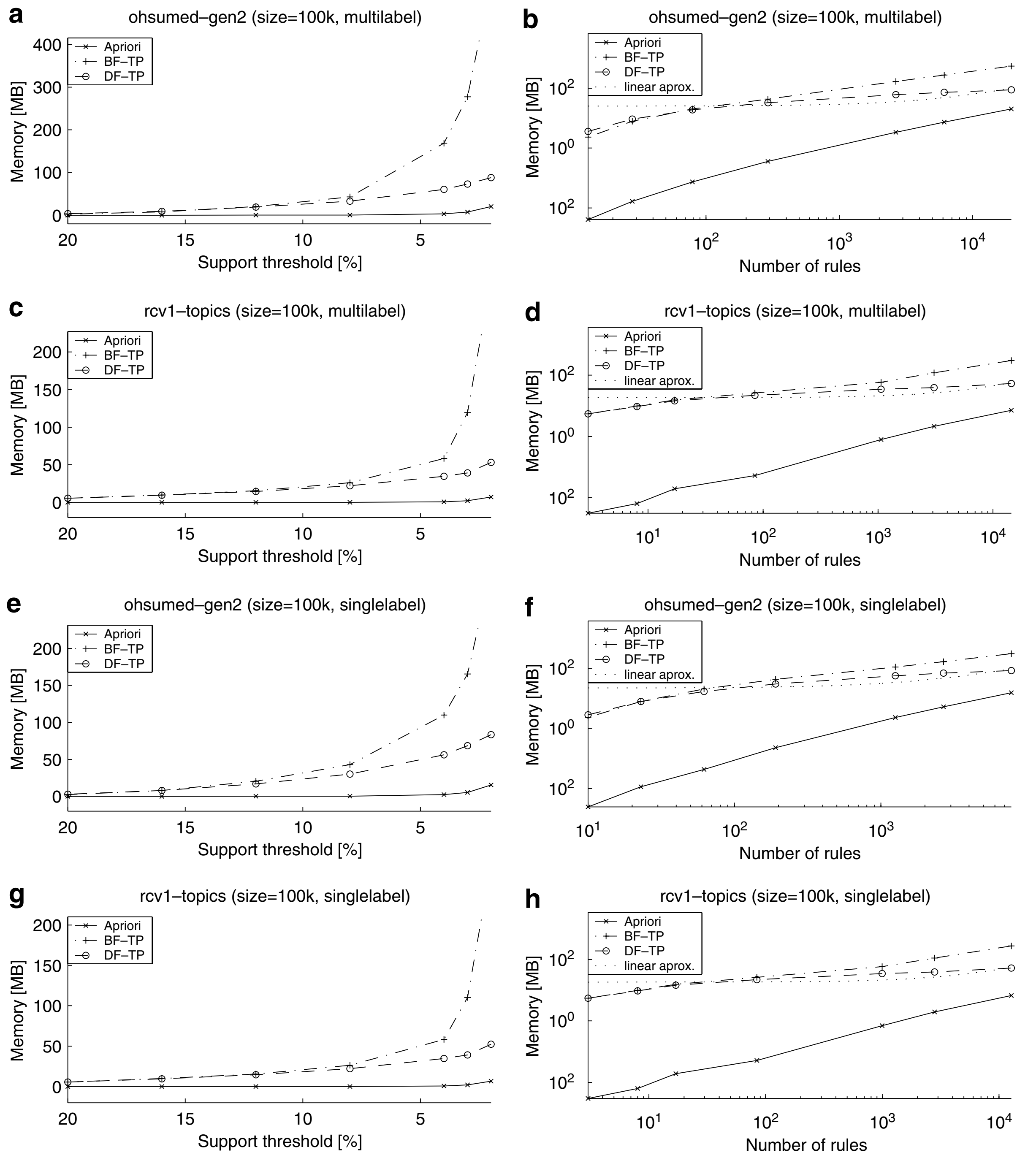


Fig. 16. Memory consumption (multi-label rules).

5.5. Single- vs. multi-label rule generation

The results discussed in the previous sections show that both single-label rule generation and multi-label rule generation preserve the same asymptotic complexity and relations between the three considered

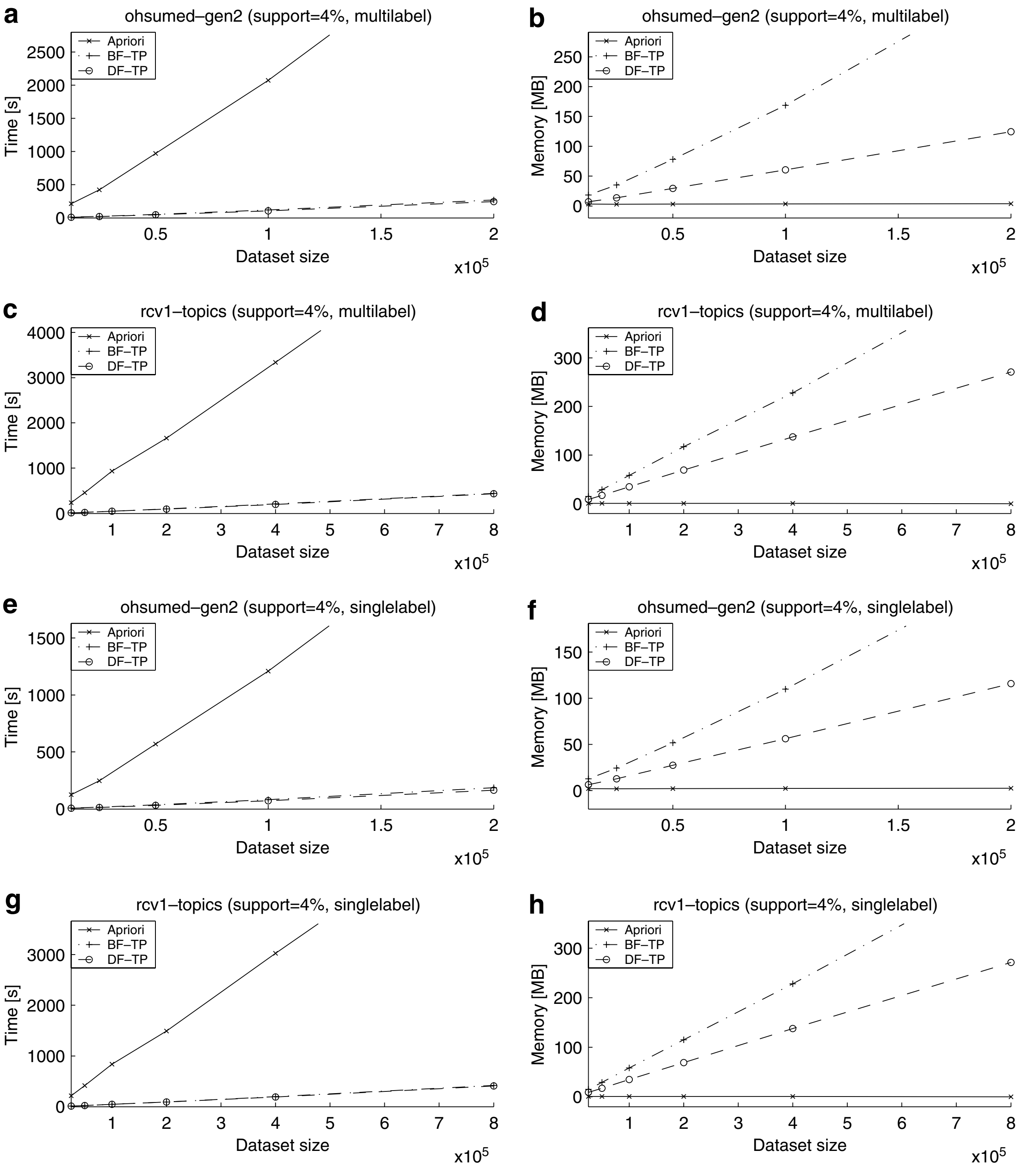


Fig. 17. Scalability (multi-label rules).

algorithms. Fig. 18 shows a summarized comparison of the cost of generating multi-label rules in relation to single-label rules with respect to the number of rules generated and runtime. For the same support threshold the two types of rule generation differ in the number of generated rules. This difference is especially visible with the low values of support threshold. For instance, setting the support threshold value to 4% results in generation of about two and a half times more multi-label rules than single-label rules with only about 80% increase

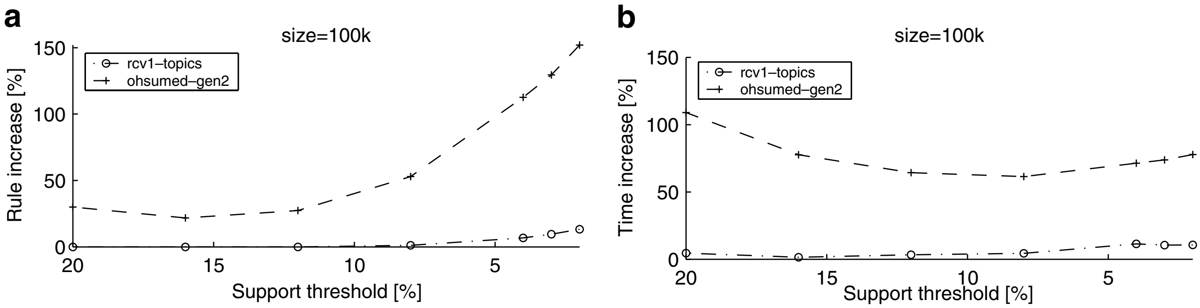


Fig. 18. Increase (a) in the number of rules between multi-label and single-label rules, and (b) in runtime to generate multi-label rules in relation to single-label rules.

in runtime. (The difference between the two datasets used in experiments is a consequence of their class label distribution, i.e., the average number of labels per transaction is significantly higher in the ohsumed-gen2 dataset than in the rcv1-topics dataset (see Table 2).) This shows that the generation of multi-label rules, that are supersets of single-label rules, is feasible with relatively low cost when compared to the generation of single-label rules.

5.6. Analysis of generated rules

Table 4 shows several rules for each of the dataset used in the experiments. All item names are shown in their stemmed form (e.g., *studi*, *compan*, and *pric*), which in some cases may become ambiguous. The presented rules were chosen in a way to show the variety of forms they can take and to be comprehensible to non-experts. There are rules with one item and one label (rules (ii), (iv), (vi), and (x)), with multiple items and one label (rules (i), (iii), (ix), and (xii)), with one item and multiple labels (rules (vii) and (viii)), and finally multiple items and labels (rules (v), (xi), and (xiii)). As expected, introducing recurrent items increases the confidence of the rule, decreasing support at the same time (compare rules (v) and (vii)). Rules with high support and low confidence (such as rule (viii)) signal that rules' items are commonly used throughout most of the classes and may be considered to be added to the list of *stop words*, i.e., the list of words that appear frequently but

Table 4
Examples of discovered rules in (a) ohsumed-gen2 and (b) rcv1-topics datasets

Rule[a]		Support (%)	Confidence (%)
(a) ohsumed-gen2			
(i)	rat(2) → Animals [B01]	5.30	99.96
(ii)	bondr → Animals [B01]	4.17	99.86
(iii)	effect, studi, control → Animals [B01]	4.29	99.35
(iv)	children → Persons [M01]	4.64	95.71
(v)	tumor(2) → Animals [B01], Neoplasms [C04]	4.11	93.80
(vi)	tumor → Neoplasms [C04]	6.13	89.09
(vii)	tumor → Animals [B01], Neoplasms [C04]	6.10	82.94
(viii)	studi → Persons [M01], Animals [B01], Investigative Techniques [E05]	12.26	30.75
(b) rcv1-topics			
(ix)	million, net, profit → Corporate/Industrial	4.02	92.28
(x)	compan → Corporate/Industrial	21.84	78.50
(xi)	net, profit → Corporate/Industrial, Performance	4.19	78.09
(xii)	net, profit → Performance	4.19	78.09
(xiii)	market, pric → Commodity markets, Markets	4.00	31.03

[a] Number in parentheses denotes recurrence.

are irrelevant to classification. Clearly, the most valuable rules, from *predictive* point of view, are those with high support and high confidence. However, in most cases such rules are of no avail from *descriptive* point of view as they usually carry obvious knowledge (see rule (x)). Thus, experts may be more interested in rules with high confidence disregarding their support at the same time (rule (ii) may be a candidate).

5.7. Classification

Although the main focus of this paper is the generation of association rules that can be used in associative-classification, we also demonstrate classification capabilities of the generated rules. We compared the classification accuracy of our method to recent results presented by Thabtah et al. [33]. That paper proposed a multiple-label associative-classification algorithm MMAC, and showed its accuracy on several benchmark datasets from the well-known UCI repository [23]. The authors compared their own MMAC algorithm to three other classifiers including a decision-tree-based algorithm PART [9], rule-based RIPPER [5], and an associative-classification algorithm CBA [20]. The four algorithms and ours alike are based on descriptive (human-interpretable) models. From the total of 19 datasets used in [33] we discarded one, namely *Autos*, since we were not able to identify an actual class attribute used by the authors.

Based on the rules generated using our algorithm (constrained by support and confidence thresholds) we applied several techniques to classify instances. For each instance a set of rules that *matched* the instance was chosen. This set of rules was further ranked according to either rules' confidence or *cosine measure* being an angle between a rule and instance represented by their vector models. The class from a rule with the highest score (biggest confidence or smallest angle) was then assigned to the instance. In case where there was no rules matching the instance, the default class was always chosen. In order to generate rules for unevenly distributed datasets we used a technique that generates rules based on a *class-size-dependent* support threshold, i.e., ruleitems are pruned based on the support threshold proportional to the size of a class they include.

Eventually, the experiments were performed using a 10-fold cross validation technique with the following parameters: (1) a support threshold, (2) a confidence threshold, (3) class-size-dependent thresholding (either switched on or switched off), and (4) either a confidence-based or cosine-measure-based rank. This set of parameters was tuned to maximize accuracy for a given dataset within 10-fold cross validation regime. Imposing cross validation ensures that the generated rules are general enough to avoid overfitting to a given training set – testing set pair. From rule expressiveness point of view it basically means less or/and shorter rules.

Table 5
Comparison accuracy of five associative-classification algorithms

	PART	RIPPER	CBA	MMAC	ACRI
Tictac	92.58	97.54	98.60	99.29	99.90
Contact-lenses	83.33	75.00	66.67	79.69	85.00
Led7	73.56	69.34	72.39	73.20	72.72
Breast-cancer	71.32	70.97	68.18	72.10	74.00
Weather	57.14	64.28	85.00	71.66	65.00
Heart-c	81.18	79.53	78.54	81.51	83.46
Heart-s	78.57	78.23	71.20	82.45	92.69
Lymph	76.35	77.70	74.43	82.20	82.24
Mushroom	99.81	99.90	98.92	99.78	95.57
Primary-tumor	39.52	36.26	36.49	43.92	51.50
Vote	87.81	87.35	87.39	89.21	94.10
Crx	84.92	84.92	86.75	86.47	86.36
Sick	93.90	93.84	93.88	93.78	93.88
Balance-scale	77.28	71.68	74.58	86.10	83.45
Breast-w	93.84	95.42	94.68	97.26	94.11
Hypothyroid	92.28	92.28	92.29	92.23	95.22
Zoo	91.08	85.14	83.18	96.15	90.00
Kr-vs-kp	71.93	70.24	42.95	68.75	66.07
Average	80.36	79.42	78.12	83.10	83.63

Table 6
Comparison of algorithms

Measurement	Algorithm	A priori	Breadth-first
Runtime	Breadth-first	++	
	Depth-first	++	~
Memory	Breadth-first	−−	
	Depth-first	−	++
Scalability – runtime	Breadth-first	++	
	Depth-first	++	~
Scalability – memory	Breadth-first	−−	
	Depth-first	−	++

+, better; ++, far better; ~, comparable; −, worse; −−, far worse.

The results for the best sets of parameters (tuned for each dataset individually) are shown in Table 5. From the total of the five presented classifiers, our classifier performed the best in terms of the average classification accuracy. Performing a paired t-test shows that the differences are statistically significant (at the 95% confidence) in all cases but one, MMAC. Won-tied-loss records of our classifier against PART, RIPPER, CBA, and MMAC are 13-0-5, 15-0-3, 13-1-4, and 10-0-8, which again confirm superior performance of the proposed method over the other classifiers. The class-size-dependent thresholding technique proved to be successful when dealing with multi-class, unevenly distributed datasets, such as *Primary tumor* (22 classes) for which our classifier outperformed others by almost 8%. The main difference in generating rules between our algorithm and the best performing other associative-classification algorithm MMAC is that MMAC iteratively generates sets of single-label rules to eventually form a set of multi-label rules, whereas the proposed algorithm creates multi-label rules at once, based on the projection tree.

6. Summary and conclusions

In this paper, we presented the design of an algorithm for generation of multi-label recurrent-item associative-classification rules. The experimental results showed superior performance of the proposed breadth-first and depth-first tree-projected algorithms over the A priori-like algorithm with respect to runtime. As expected, memory consumption is bigger for the proposed tree-projected algorithms than for the A priori-like algorithm due to usage of projected datasets to generate rules. Although this difference is significant for the breadth-first search, the memory characteristics indicate significantly better (smaller) memory consumption for the depth-first algorithm. The comparison of the three algorithms is summarized in Table 6. The comparison shows that the proposed depth-first search multi-label association classification rule generation algorithm provides the best balance by being characterized by low runtime and moderate memory consumption. The proposed method is the first to address generation of multi-label association rules and achieves performance that is comparable to a single-label rule generation. In short, we showed that the added value of multi-label rules comes with a reasonably low cost in terms of both runtime and memory consumption. The algorithms scale linearly with respect to the dataset size for both runtime and memory usage. Although complexity with respect to the number of items may become exponential (for frequent and long patterns), we showed several tree-based techniques to significantly slow down this growth, which makes our algorithms, and especially the depth-first strategy, more efficient than those of other researchers.

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