Learning fuzzy cognitive maps with required precision using genetic algorithm approach

W. Stach, L. Kurgan, W. Pedrycz and M. Reformat

Fuzzy cognitive maps (FCMs) are a powerful and convenient tool for describing and analysing dynamic systems. Their generic design is performed manually, exploits expert knowledge and is quite tedious, especially in the case of larger systems. This shortcoming is alleviated by completing the design of FCMs through learning carried out on experimental data. Comprehensive experiments reveal that this approach helps design models of required accuracy in an automated manner.

Introduction: Fuzzy cognitive maps (FCMs), introduced by Kosko [1], represent a given dynamic system as a set of concepts and mutual relationships among them. The FCM models were developed and used in numerous research and industrial areas, such as electrical and software engineering, supervisory systems, medicine, political science, military science, history etc. The main drawback of FCMs is related to their manual, based on expert knowledge, and relatively complex development phase. Therefore only small models are being developed. A few methods for automated or semi-automated learning of FCMs have been proposed to date [2]. So far, for no method has convergence been demonstrated [3]. Recently, we developed and comprehensively tested on synthetic, randomly generated, FCM models a new genetic algorithm (GA) based approach to the automated learning of FCMs. The new method is able to learn the model from data, and therefore substitutes for the expert. This Letter investigates the hypothesis that increasing the size of input data leads to improving the quality of the learned FCM model, and executes comprehensive tests on several new published models.

![Diagram of proposed learning method](image)

**Fig. 1 Diagram of proposed learning method**

--- Typical application of proposed learning method
--- Test procedure

The Figure should be read starting at the bottom left corner and following in clockwise manner.

Learning method: The proposed learning method develops an FCM, called the candidate FCM, from input data using a genetic algorithm, see Fig. 1. Given a system consisting of $N$ concepts, the FCM model can be described fully by its connection matrix, which consists of $N(N-1)$ variables assuming values in $[-1, 1]$ [1]. Simulation of an FCM consists of computing the new state of the system, described by a state vector, from a previous state of the system, over a number of successive iterations. The state vector specifies values of all concepts in a particular iteration. The learning uses experimental data consisting of consecutive state vectors obtained for different initial conditions. Each two successive states of the system from any state vector can be used during learning. Defining $C(t)$ as an initial vector and $C(t+1)$ as system response (i.e. next state), the input data can be expressed as a set of (initial vector, system response) pairs. The number of these pairs determines the size of the training data. A fitness function that constitutes the core of the genetic algorithm is defined as a normalised error between state vectors generated by a candidate FCM and the experimental data. Different fitness functions based on $L_1$, $L_2$ and $L_\infty$ norm have been tested; we found that $L_2$-norm provides the best convergence. Other parameters of the GA, such as population size, number of generations, mutation mechanisms etc., were established experimentally to optimise the performance of learning for FCM models of different sizes and levels of mutual relationships.

Experiments: To demonstrate that the proposed learning method is valid and accurate, a comprehensive set of tests on diverse, published in the literature, FCM models was performed. Because of the manual development method, those FCMs are relatively small and consist of about five, and no more than ten, nodes. The experiments involved two models composed of five nodes: control process (referred later as Model 5.1) [4], heat exchanger (Model 5.2) [5]; two eight-node models: control process (Model 8.1) [6], European Monetary Union and the risk of war (Model 8.2) [7]; and two models that consist of ten nodes: virtual squad of soldiers (Model 10.1) [8], and health issues (Model 10.2) [9]. The experiments used input data generated from or reported in the corresponding References.

Quality of the generated candidate FCM is evaluated according to several criteria. In general, the only available information about the modelled system is the input data, see Fig. 1. Thus, the first criterion measures the difference between data generated by the candidate FCM and the given input data set. Normalised least square error measure is reported as error-$L_2$. However, knowledge of the original FCM model allows for more detailed and richer comparison considering the difference between the models. An average absolute difference between corresponding entries in the connection matrices is reported here as error-weight. Learning precisely the connection matrix is crucial, since most relationships, expressed by its cell values, come with some meaning. Experiments aim to analyse the relationship between the input data size (length) and the quality of solution for different FCM model sizes.

![Error-$L_2$ against input data size](image)

**Fig. 2 Error-$L_2$ against input data size**

Results: Experiments reveal that the proposed method produces high quality FCM models both in terms of similarity to those proposed by experts (expressed by the connection matrix), and quality of fitting the input data. Fig. 2 shows error-$L_2$ against input data size for the six models. The models generated for small input data sizes resulted in small error-$L_2$ values, but large error-weight, see Fig. 3. This indicates that, because input data were short, they can be generated accurately by different FCM models. We note that resulting candidate FCM will correctly generate the input data, but at the same time it may not correctly generalise knowledge about the underlying model, which is expressed by the connection matrix, i.e. it will generate an incorrect state vector for a different initial condition. Increasing the input data size initially leads to slight degradation of the quality of fitting input data, but at the same time the error-weight decreases. This shows that it is harder to approximate larger input data when multiple models provide accurate approximation, but the
learned solutions start to converge to an optimal solution. By further increasing the size of input data, both error-$L_2$ and error-weight converge to zero, which indicates that only one solution exists for longer input data, and the generated candidate FCM is identical to the expert’s model.

![Graph showing error-weight against input data size](image)

**Fig. 3** Error-weight against input data size

The second finding shows the relation between the size of input data size and the convergence of the candidate FCM to the desired (original) model, see Fig. 3. We note that insufficient size of input data results in high value of error-weight, and therefore the generated model is substantially different from the desired solution. By increasing size of available data, the candidate FCM converges to the desired solution with user-required accuracy. In general, the larger the number of concepts in the model, the larger the size of input data is necessary to achieve high accuracy of the model. We stress that these observations are valid throughout all models considered in this study.

**Conclusions:** Overall, the proposed learning method led to promising results. Three important conclusions can be drawn: (1) given input data of sufficient size, the method can generate FCM models that are identical to models proposed by domain experts; (2) increasing size of input data improves accuracy of learning; and (3) multiple different models can be generated from input data of small size, and therefore insufficient size of input data may result in poor quality of learning. We note that the developed method constitutes an important milestone towards eliminating human intervention in developing FCM models.

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**References**